Vibration Analysis of Functionally Graded Carbon Nanotube Reinforced Beam Structures

Thesis submitted in partial fulfillment of

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Bachelor of Technology in Mechanical Engineering

By

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CERTIFICATE

This is to certify that the thesis entitled, "**Vibration Analysis of Functionally Graded Carbon Nanotube Reinforced Beam structures**" submitted by Mr. Nitish Kumar (111ME0304) in partial fulfillment of the requirements for the award of Bachelor of Technology in Mechanical Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by them under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

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ABSTRACT

This work deals with the study of vibration behavior of the Functionally Graded Timoshenko Beam that has been reinforced with Carbon Nanotubes (CNTs), which is subjected to thermal and mechanical loads. The constituent materials of the functionally graded beam are alumina as the matrix material and single walled CNTs as the reinforcement material. The volumetric fraction varies according to power law along the thickness. The temperature dependent (TD) and temperature independent (TID) material properties of the beam are determined by employing Mori-Tanaka method and extended rule of mixture along the thickness direction. Timoshenko beam theory is used to study the dynamic behavior of the beam. The finite element method is employed to discretize the model and Hamilton's principal is used to derive the equation of motion . Vibration analysis has been carried out to study the response of Temperature dependent and independent material properties on the dynamic behavior of the beam. The results show that CNT volume fraction and Temperature dependent material properties has substantial effect on the vibration characteristic of the beam.

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NOMENCLATURE

Although all the principal symbols used in this thesis are defined in the text as they occur, a list of them is presented below for easy reference.

V_{cnt}	Volume fraction of ceramic (Al_2O_3)
V_m	Volume fraction of ceramic (Al_2O_3)
z	Z axis
h	Thickness of beam
k	Power law index
K	Thermal conductivity of the material
$E(z)$	Effective young modulus of FG-beam at a layer in z coordinates.
E_{cnt}	Young modulus of Carbon Nanotubes
E_m	Young modulus of matrix, which is the ceramic material Al_2O_3
$A0, A1, A2, A3, A_{-1}$	Constants in cubic fit of the material property
T_m	Temperature of matrix
T_{cnt}	Temperature of CNT
κ_{cm}	Difference between Conductivity of CNT and ceramic material
κ_m	Conductivity of the ceramic material (Al_2O_3)
κ_{cnt}	Conductivity of the CNT
$A(T)$	Physical properties with respect to temperature

E	Young's Modulus
ρ	Density
ν	Poisson's ratio
b	Width of the beam
h	Thickness of the beam
E_{cm}	Difference between Young's modulus of CNT and ceramic
C	Effective elastic modulli
E_{11}^{cnt}	Longitudinal young modulus of CNT
E_{22}^{cnt}	Transverse young modulus of CNT
G_{12}^{cnt}	Shear modulus of CNT
C_m	Stiffness tensor of matrix
C_r	Stiffness tensor of equivalent fiber
A_r	Dilute mechanical stress concentration
[M]	Mass matrix
[K]	Stiffness matrix

Chapter 1

Introduction

INTRODUCTION

1.1. Composites

The word composite refers to combination of two or more materials to form a useful other material on a macroscopic scale. The advantage of composite materials is the exhibition of superior properties possessed by their components. By making a composite material, the properties can be varied to a greater extent. These properties are:

i. Strength
ii. Stiffness
iii. Temperature dependent behavior
iv. Thermally insulated property
v. Thermal properties ,and
vi. Fatigue life

Composite material can be used for structure application .The structures which are sensitive to weight can use composite material and in this way it can become cost effective. Some of the types of vastly used composites are fiber reinforced polymers, metal composites, metal-ceramic. The homogeneity decides the classification of composites as simple composite and functionally graded composites. . Any elementary section of the composite would have same volumetric composition as any other section. Because of this all the physical properties are same throughout its volume. The compositions of the constituent materials vary according to a predefined mixing rule in a functionally graded material. The mixing could be exponential or follow a power law. The volumetric composition of different point will be different and the properties will vary accordingly.

1.2. Carbon Nanotubes (CNTs)

Allotropic forms of carbon with a cylindrical nanostructure are carbon nanotubes. The synthesis of carbon nanotubes includes various process such as include arc discharge, high-pressure carbon monoxide disproportionation, laser energy, and chemical vapor deposition (CVD).The construction of nanotubes includes high diameter to length ratio up to

1:132,000,000,significantly larger than any other material. Because of their extraordinary physio-chemical properties, CNTs can be used as best materials for energy, biomaterials, optoelectronics and defense. The tremendous efforts has been going on to produce economical methods for CNTs production, the main problems with this are quality of structure and increment in purity, scalability and control over chirality or control over diameter. The factors behind its superior properties are the topological arrangement of the atom in the CNT and its overall tubular structure. The carbon atoms are bonded like hexagonal network which is periodic represents tubular structure which is in form of lattice are carbon nanotubes .A single sheet consisting of graphite can be rolled up to generate tubular structure which can be easily visualized while the end caps which are like fullerene are present at the ends of the tube. The atoms of carbon in a nanotube are arranged in a network which is hexagonal represents the configuration which is toroid.

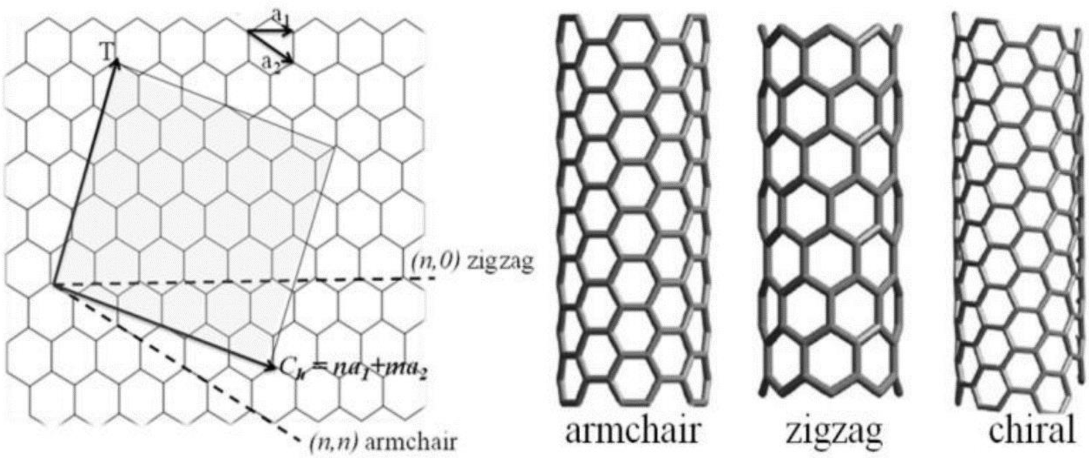

Figure 1. The construction of CNTs from a graphene sheet along the chiral vector C_h [1].

1.3. Functionally Graded Materials(FGMs)

Japan Aerospace exploration Agency (JAXA) is the first to use FGMs in the 1980s.This leads to the motivation towards research in the field of graded composites all around the world because of its benefits. The composites of two or more materials with gradually varying composition of the constituents is referred to Functionally Graded Material. Various functions such as exponential function or power law can mathematically control this variation. The difference in the material properties in the material rises owing to volumetric variation. During present days, functionally graded material (FGMs) have become significantly important in extreme conditions of temperature. These type of environment exists in nuclear reactors and chemical plants. Functionally graded materials can be used as the best material for the upcoming high-speed aircraft. Functionally graded materials are the composites which are inhomogeneous in nature. These composites properties as well as composition vary smoothly and continuously along the thickness direction and as a result of this can be used for wide range of applications in many industries. This can be done by varying the volume of the constituents along the thickness direction. In this way the properties of FGMs which are graded are a result of continuous change in composition. These materials have advantages that the environments which consists of high-temperature gradient can be withstood easily by them in order to maintain integrity of structure. The initial design of FGMs in materials which acts as thermal barrier are used in aerospace and structural applications. The better thermal stress capacity is because of ceramic and better mechanical load bearing capacity because of the presence of metal will be there in an FGM of a ceramic and a metal .The failures which can be possible owing to thermo or both thermo-mechanical loadings in environments consisting of high temperature can be reduced by these functionally graded structural element.

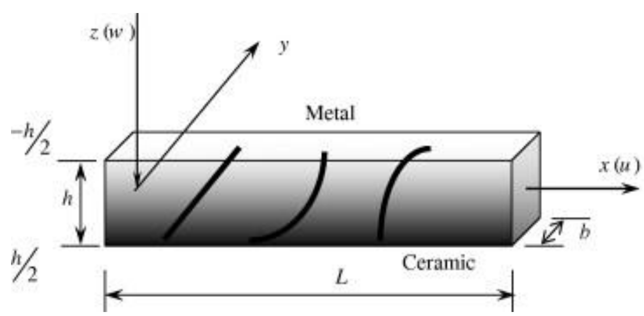

Figure 2: Schematic of FG beam with used coordinates [2]

1.4. *Applications of FGM*

1. Aviation and aerospace
    - Outer parts of aerospace
    - Engine parts of rocket
    - Vibration control
    - Structures having an ability to change to suit to different conditions

2. Design and engineering
    - Tools used for cutting material
    - Engine wall linings
    - Different types of shafts
    - Different components of engine
    - Blades of hydraulic machines such as turbines
3. Nuclear energy
    - Components of nuclear reactor
    - Fusion reactor wall
    - Pellet of fuel
4. Optics
    - Optical lens
    - Fibre of optics
5. Electronics industry and plant
    - Semiconductor of graded band
    - Sensor device
    - Integrated chips
6. Chemical industry
    - Different types of heat exchanger and heat pipes
    - Different types of reaction vessels

1.5 Outline of present work

This present work deals with the vibration analysis of the Functionally Graded Timoshenko Beam that has been reinforced with Carbon Nanotubes (CNTs), which is subjected to thermal and mechanical loads. The constituent materials present in the functionally graded beam are alumina (Al2O3) as the matrix material and single walled CNTs as the reinforcement material. The volumetric fraction of constituents varies according to power law along the thickness. The temperature dependent and temperature independent material properties of the beam are determined by employing Mori-Tanaka method and extended rule of mixture along the thickness direction. Timoshenko beam theory is used to study the dynamic behavior of the beam. The finite element method is employed to discretize the model and Hamilton's principal is used to derive the equation of motion. Free Vibration and transient response analysis has been carried out to study the response of Temperature dependent and independent material property on the dynamic behavior of the beam.

Chapter 2

Literature Review

LITERATURE REVIEW

The mechanical, electrical and thermal properties can be improved by the addition of carbon nanotubes into composite structures. Therefore structures containing CNT are very useful in distinctive types of industry for their multi -functionalities. Diverse hypotheses have been proposed over years and new method of solution is have been produced. The extensive significance of functionally graded materials (FGMs) has been expanded over years in extremely high temperature conditions for example, atomic reactor and synthetic industries. The composite structures have been useful for different types of roles in different applications of engineering because of superior properties as compared to other materials. They are highly sensitive towards any properties which make them useful in different conditions for various purposes. Murmu and Pradhan [3] applied the nonlocal elasticity theory to investigate the free vibration pro- blem of nanoplate's under uniaxially prestressed conditions. In their study the differential quadrature method was utilized to obtain the fundamental natural frequencies for simply supported and clamped nanoplates. The place where the failure is initiated is the sharp interfaces present in composite material taken out by functionally graded material [4]. The sharp interfaces defined above are replaced by a gradient surface due to which there is smooth change from one to another[5,6]. Another attributes of FGMs is the capacity to prepare a material for specific application [7]. Functionally Graded Material, a progressive material, fits in within the category of advanced material with the properties which are fluctuating in accordance with changing dimension [7,8]. These materials are present in the nature in the form of teeth and bones [9], these materials are composed and designed by nature to meet their expected service prerequisites. As simulated neural system copy human mind, in the same way building issues are illuminated by the thoughts copied from the nature.

A Chakraborty et al. [10] have studied about the first-order deformation theory and explained the behavior of functionally graded beam structures which is thermo-elastic and these attributes and properties are varying along its thickness. Interpolation of polynomials for the element formulation can be constructed by the governing differential equations. The various types of stresses were observed and determined using various functions such as power and exponential law. Thermal behaviors of these beam which is functionally graded (FGB) by taking the distribution of the attributes of material in several functions such as exponential function were analyzed and examined by GH Rahimi and AR Davoodinik [11].These variations in form of exponential and hyperbola in the thickness direction of heat conduction in steady state were considered for use of thermal loading. They found that thermal behavior of both isotropic beam and functionally graded beam depend up on the temperature distribution. Noda [12] bestowed a review covering various topics which is from themoelastic to themoinelastic issues. He examined impact of mechanical properties which are temperature dependent on stress .He recommended that these properties of the material which are temperature dependent should be taken into account to carry out precise analyses. Tanigawa [13] incorporated detailed discussion of these models analytically in order to explain thermo elastic behavior of FGMs. In the discussion, he presented the idea of solutions which are closed form for the problems involving heat conduction. B.V. Sankar [14] has explained an elasticity solutions for beam which is functionally graded subjected to transverse loads. The solution for these equations were obtained by the exponential change in the elastic stiffness coefficient. T. Prakash et al. [15] have examined post buckling action of skew plates functionally graded to which thermal load is applied .Mohammad Azadi [16] has read thoroughly about the method known as finite element method. To determine different natural frequencies of beams which is functionally graded, with different boundary conditions. He compared his results with the solution obtained analytically with software such as NASTRAN and ANSYS. These numerical results determine consequence of material properties which are temperature dependent, volume fraction distribution, geometrical parameters and boundary conditions. The stress intensity because of temperature variation producing crack in a material was minimized by Jin and Noda [17].Arani et al. [18] were together analyzed the stress due to temperature difference of a cylinder reinforced with functionally graded carbon nanotubes. Based on Mori-Tanaka method, thermal field was subjected to thin- walled cylinder. The nano-composite characteristics which is transversely isotropic was applied on uniformly distributed SWCNTs. The distribution of displacement and thermal stresses in radial, circumferential and axial directions was obtained by solving higher order governing equation whose results indicate that the significant effect on thermal stresses and displacements in axial, radial and circumferential directions is because of FG distributions of SWCNTs. Analytical thermo-elastic stresses expressions were derived by K. Nirmala et al. [19] discussed numerical analysis of discretization of the beam and FGM layer which is continuous can be treated as a discretely graded structure .Heshmati and Yas [20] have analyzed and discussed idea of properties which are because of vibration in a nano-composite beam functionally graded with single walled carbon nanotubes subjected to moving load. These properties were explored utilizing Eshelby–Mori–Tanaka approach. The thermo-mechanical dynamic traits of SWCNT-Reinforced Composite Plates was studied by A Shooshtari et al. [21]. Using the multi-scale process, numerical values were determined for CNTRC plates and uniformly distributed CNTRC plates. The results predicted the reduction in natural frequencies .Wang et al. [22] have analyzed the consequences of environmental and natural temperature on properties of single walled carbon nanotubes which is elastic using the method of molecular structural mechanics. Murmu and Adhikari [23] applied the differential quadrature method and the nonlocal elasticity theory to study the free vibration of a rotating carbon nanotube modeled as an Euler–Bernoulli beam. Duan and Wang [24] obtained exact solutions for the axisym- metric bending of micro- and nano-circular plates under general loading using a nonlocal plate theory. He explained that bending stiffness is affected by nonlocal parameter. Civalek and Demir [25] applied the differential quadrature method to study the bending analysis of microtubules subjected to uniformly distributed and concentrated loads. Civalek and Akgoz [26] applied the nonlocal elasticity theory to study the free vibration analysis of micro-scaled annular sector and sector shaped graphene located on an elastic matrix. In this method ,frequency parameter was obtained by a method known as discrete singular convolution. Y. Kiani and M.R. Eslami [27] have studied about buckling in beam in which thermo-mechanical loading is applied and the assumption was that nonhomogeneous properties changes smoothly according to some function known as power law across the thickness of the beam. G.R. Liu et al. [28] have introduced a model in which there is absence of lattice for thermodynamic shape variation.The element-free Galerkin method derived the shape functions with the help of the moving least squares method.

Chapter 3

Motivation

And

Objectives

MOTIVATION AND OBJECTIVES

The functionally graded materials have the advantage of reducing stress concentration and delamination found in composite structures. The addition of CNT in to FGM not only improves the thermal and mechanical properties but also helps to reduce vibration of the structure. Further in applications where temperature plays major role it is important to study the effect of temperature on the material properties and consequently their effect on the dynamic behavior of the structure. Meanwhile the metals being used are responsible for increasing the weight of the structure. Therefore the excellent choices are CNTs to replace metals in the FG composite structure. Based on these the present analysis is carried out and it will be helpful for predicting the dynamic behavior of the structures for distinct applications.

The objectives of the present work are:

i. Mathematical modelling for material properties of the FG-CNTRC beam.
ii. To determine the temperature dependent and temperature independent material properties
iii. Finite element modelling and analysis of functionally graded beam structures and,
iv. Modal analysis and impulse response of such structures

Chapter 4

Methodology

METHODOLOGY

The mathematical modelling of beams have been evaluated using power law function of the FGM (the volume of constituents vary along thickness direction) and applying the various physical properties like Young's Modulus and other properties such as coefficient of thermal expansion and Thermal conductivity of the ceramic material (Al203) and the CNTs. The properties variation and temperature variation have been obtained using relations of property in accordance with temperature and volume fraction.

4.1 Material modelling

In the present analysis the constituents of the composite considered are alumina as a matrix phase and SWCNT as reinforcement phase. The material modelling is carried out as mentioned below:

4.1.1. Rule of mixture

The modelling of FGM can be done using various functions for distribution of constituent material in a beam; these are termed as rule of mixture. In exponential law the volume fraction of one element increases and other component decreases exponentially. A particular index has been used for constituents in a power law. The power law has been used for this analysis.

4.1.2 Power law material distribution

The material properties can be calculated using power law in which volume of both constituents vary in the thickness direction. The fraction of volume changes in accordance with power law in the thickness direction. It is given according to following equation [29]

$$V_{cnt} = \left(\frac{z}{h} + 0.5\right)^{n} \tag{1}$$

Here n commonly known as the power law index, *h* represents thickness and *z* is co-ordinate which varies from (– $h/2 \le z \le h/2$), along the thickness direction. As a result, the required material properties will be [29]

$$E(z) = E_m + E_{cm} V_{cnt}, \quad k(z) = k_m + k_{cm} V_{cnt} \quad , \quad \alpha(z) = \alpha_m + \alpha_{cm} V_{cnt} \qquad (2)$$

Where these subscripts cnt and m refer to the CNT constituents and matrix respectively and are related as [29]

$$E_{cm} = E_{cnt} - E_m \,, \quad \alpha_{cm} = \alpha_{cnt} - \alpha_m \,, \quad k_{cm} = k_{cnt} - k_m \qquad (3)$$

In Eqs (2), E represents young modulus and α is thermal expansion coefficient and k is thermal conductivity. The materials properties are calculated for each layer of beam and based on these properties further calculations are determined.

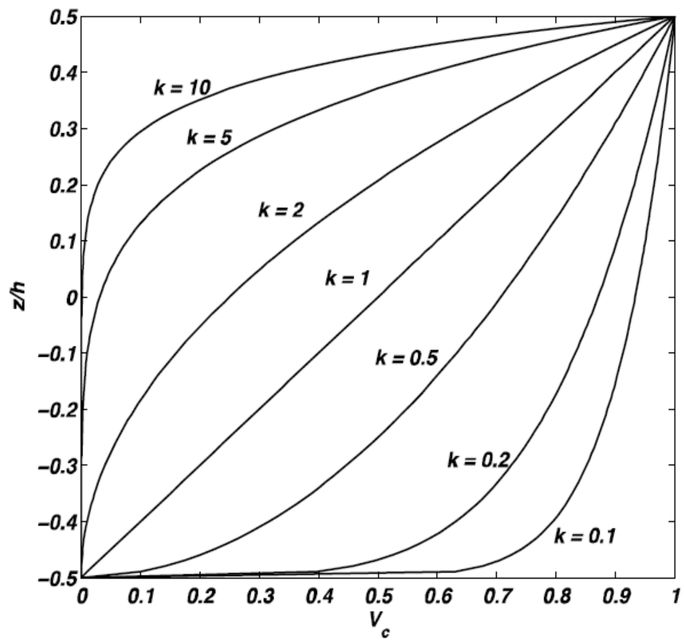

Figure3: Distribution of ceramic volume for various power index in the thickness direction [30]

4.1.3 Extended rule of mixture

The extended rule of mixture defines effective material properties of beam structures in the following manner:

Where, E_{11}^{cnt} represents young modulus in longitudinal direction and E_{22}^{cnt} represents young modulus in transverse direction. G_{12}^{cnt} denotes shear modulus of CNT. E_m and G_m denote the properties of matrix respectively. η_i (i=1,2,3) are called efficiency parameters. V_{cnt} and V_m represent volume of carbon nanotube and volume of matrix respectively.

4.1.4 Mori-Tanaka method

Various methods can be used to determine the composite properties. The Mori-Tanaka method is a very simple and accurate method. The present study uses Mori-Takana method to predict elastic stiffness. The Eshelby-Mori-Tanaka method is based on the idea of and Mori-Tanaka. Eshelby used the idea of equivalent elastic inclusion and Mori-Tanaka used the idea of average stress in matrix. The Benveniste's revision gives tensor of effective elastic moduli C of CNTRC. [31]

$$C = C_m + V_{cnt} \left\langle \left(C_r - C_m\right) A \right\rangle \left(V_m I + V_{cnt} \left\langle A_r \right\rangle\right)^{-1}$$

Where I represents fourth-order unit tensor. C_m and C_r is stiffness tensor of matrix and the stiffness tensors of the equivalent fiber. The brackets represents an average possible orientation of inclusions. A_r is the dilute mechanical stress concentration and is as follows: [31]

$$A_r = \left[I + S\left(C_m\right)^{-1}\left(C_r - C_m\right)\right]^{-1}$$

Where s is fourth-order Eshelby-tensor. The hill's elastic modulli of the reinforcing phase can be calculated as [31]

$$C_r = \begin{bmatrix} n_r & l_r & l_r & 0 & 0 & 0 \\ l_r & k_r + m_r & k_r - m_r & 0 & 0 & 0 \\ l_r & k_r - m_r & k_r + m_r & 0 & 0 & 0 \\ 0 & 0 & 0 & p_r & 0 & 0 \\ 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & p_r \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{LEF}} & \frac{-\upsilon_{EF}}{E_{LEF}} & \frac{-\upsilon_{EF}}{E_{LEF}} & 0 & 0 & 0 \\ \frac{-\upsilon_{EF}}{E_{LEF}} & \frac{1}{E_{LEF}} & \frac{-\upsilon_{EF}}{E_{LEF}} & 0 & 0 & 0 \\ \frac{-\upsilon_{EF}}{E_{LEF}} & \frac{-\upsilon_{EF}}{E_{LEF}} & \frac{1}{E_{LEF}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{EF}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{EF}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{EF}} \end{bmatrix}^{-1}$$

k_r, l_r, m_r, n_r, and p_r are hill,s elastic modulli for the reinforcing phase(CNTs)of the composite. The two Euler angle α and β represents orientation of straight CNT as shown in the figure.

4.2 Temperature distribution

It is obtained by using FDM. Firstly, the material properties which vary in accordance with temperature are assumed according to the polynomial series as follows [30]

$$A = A_0\left(A_{-1}T^{-1} + 1 + A_1 T + A_2 T^2 + A_3 T^3\right) \tag{4}$$

Where, A_0, A_{-1}, A_1, A_2 and A_3 are the constants of coefficients of temperature. These are constants for different types of materials for a range of temperature which is fixed. Now, One-dimensional Fourier conduction heat equation is as follows [30]

$$\frac{d}{dz}\left[k(z)\frac{dT}{dz}\right] = 0 \tag{5}$$

Where, k is the thermal conductivity and T is the temperature. The pure ceramic layer is subjected to a temperature of 700 K, whereas the pure CNT layer is subjected to a temperature of 1000 K. The boundary conditions for temperature are [6] as follows

$T = T_m$ =700 K *at* Bottom

$T = T_{cnt}$ =1000 K *at* Top

The profile of temperature in the beam in the thickness direction is given as follows [29]

$$T(z) = T_m + (T_{cnt} - T_m)\eta(z) \tag{7}$$

Where,

$$\eta(z) = \frac{1}{C}\left[\begin{array}{l}\left(\frac{2z+h}{2h}\right) - \frac{k_{cm}}{(n+1)k_m}\left(\frac{2z+h}{2h}\right)^{n+1} + \frac{k_{cm}^2}{(n+1)k_m^2}\left(\frac{2z+h}{2h}\right)^{2n+1} - \frac{k_{cm}^3}{(n+1)k_m^3}\left(\frac{2z+h}{2h}\right)^{3n+1} \\ + \frac{k_{cm}^4}{(n+1)k_m^4}\left(\frac{2z+h}{2h}\right)^{4n+1} - \frac{k_{cm}^5}{(n+1)k_m^5}\left(\frac{2z+h}{2h}\right)^{5n+1}\end{array}\right]$$

$$C = 1 - \frac{k_{cm}}{(n+1)k_m} + \frac{k_{cm}^2}{(n+1)k_m^2} - \frac{k_{cm}^2}{(n+1)k_m^2} + \frac{k_{cm}^4}{(n+1)k_m^4} - \frac{k_{cm}^5}{(n+1)k_m^5} \;;$$

Eqs. (4) gives temperature distribution along thickness

Table 1: Temperature-dependent material properties for (10, 10) SWCNT [32]

Temperature (K)	E_{11}^{cnt} (TPa)	E_{22}^{cnt} (TPa)	G_{12}^{cnt} (TPa)
300	5.6466	7.0800	1.9445
500	5.5308	6.9348	1.9643
700	5.4744	6.8641	1.9644
1000	5.2814	6.6220	1.9451

Table 2. The constants in the cubic fit of Alumina

Properties	***A0***	***A-1***	***A1***	***A2***	***A3***
Modulus of Elasticity(GPa)	349.55	0	-3.853×10^{-13}	4.027×10^{-16}	1.673×10^{-10}

Poisson's ratio	0.3000	0	0	0	0
Coefficient of Thermal expansion (K^{-1})	6.8269×10^{-6}	0	1.838×10^{-4}	0	0
Thermal Conductivity(W/mK)	-14.0287	-1123.6	-6.227×10^{-3}	0	0

Table 3. Constants in Cubic fit for Young's Modulus of (10, 10) SWCNT

(*L* = 9.26 nm, *R* = 0.68 nm, *h* = 0.067 nm, □$_{CN}^{12}$ =0.175)

Properties	***A0***	***A-1***	***A1***	***A2***	***A3***
Modulus of Elasticity(TPa)(E11)	6.1515	0	-4.38×10^{-4}	6.31×10^{-7}	-3.40×10^{-10}
Modulus of Elasticity(TPa)(E22)	7.7132	0	-4.27×10^{-4}	6.31×10^{-7}	-3.40×10^{-10}
Coefficient of Thermal expansion (K^{-1}) $\times10^{-6}$	-1.1209	0	-0.02043	2.57×10^{-5}	-1.01×10^{-8}
Shear modulus (G12)(TPa)	1.8604	0	2.227×10^{-4}	-2.666×10^{-7}	8.952×10^{-11}

4.3 *Free vibration analysis of FG-CNTRC beam structures using FEM*

According to first order shear deformation theory, the axial displacement u and transverse displacement w at any point in the beam are given by [31]

$$u(x,y,z,t) = u_0(x,t) - z\phi(x,t)\ , \quad w(x,y,z,t) = w_0(x,t) \tag{8}$$

The strain energy and kinetic energy are determined by strain-displacement and constitutive relations. The expressions for these energies are given below:

$$U_b = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz})dAdx; \quad T_b = \frac{1}{2}\int_v (\dot{u}^2 + \dot{w}^2)dAdx \tag{9}$$

The differential equations with three degree of freedom which consists of two translational and one rotational can be carried out by applying Hamilton's principle i.e. u, w and ϕ [31]

$$\begin{aligned}
&\delta u : I_0(x)\ddot{u} - I_1(x)\ddot{\phi} - \frac{\partial}{\partial x}\left(A_{11}(x)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x}\left(B_{11}(x)\frac{\partial \phi}{\partial x}\right) = 0 \\
&\delta w : I_0(x)\ddot{w} - \frac{\partial}{\partial x}\left(A_{55}(x)\frac{\partial w}{\partial x} - \phi\right) = F_0\delta(x - vt) \\
&\delta\phi : I_2(x)\ddot{\phi} - I_1(x)\ddot{u} + \frac{\partial}{\partial x}\left(B_{11}(x)\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial x}\left(D_{11}(x)\frac{\partial \phi}{\partial x}\right) - k_s A55(x)\left(\frac{\partial w}{\partial x} - \phi\right) = 0
\end{aligned} \tag{10}$$

The force boundary conditions are given by [31]

$$A_{11}\frac{\partial u^0}{\partial x} - B_{11}\frac{\partial \phi}{\partial x} = N_x\ ,\ A_{55}\left(\frac{\partial u^0}{\partial x} - \phi\right) = V_x\ ,\ \text{-B}_{11}\frac{\partial u^0}{\partial x} + D_{11}\frac{\partial \phi}{\partial x} = M_x \tag{11}$$

Where N_x represents axial force, V_x represents shear force and M_x represents bending moment which is given and acting at boundary nodes .As a result the stiffness coefficient can be determined as follows[31]

$$\left[A_{11}\ \ B_{11}\ \ D_{11}\ \right] = \int_A E(z)\ \left[1\ \ \text{z}\ \ \text{z}^2\right] dA\ ,\ \left[A_{55}\right] = \int_A G(z)\ \text{dA} \tag{11}$$

And the mass moments are [31]

$$\left[I_0\ \ \text{I}_1\ \ \text{I}_2\ \right] = \int_A \rho(z)\ \left[1\ \ \text{z}\ \ \text{z}^2\right] dA \tag{12}$$

4.3.1 Finite element analysis

The implementation of finite element analysis has been performed to solve above governing equation. This method uses approximate analysis to carry out displacement field variable at nodal points. The interpolation functions of the displacement field for the FEM formulation are determined by solving governing differential equations. These solution are represented as follows which is given below [31]

$$u^0 = c_1 + c_2 x + c_3 x^2 \ , \ w^0 = c_4 + c_5 x + c_6 x^2 + c_7 x^3 \ , \ \phi = c_8 + c_9 x + c_{10} x^2 \quad (13)$$

It is easily seen the order of slope ϕ is one order lesser than the order of w^0. After solving the governing equations using the interpolation functions, the stiffness and mass matrix can be obtained. Finally, the derived equation of motion is as follows

$$[M]\left\{\ddot{q}\right\} + [K]\{q\} = \{F\} \quad (14)$$

Where $[M]$ represents the mass matrix, $[K]$ represents the stiffness matrix

.

4.4 Transient response due to impulse loading

Impulse loading is applied at middle node and the constant load is sustained up to 10 time step. It is very difficult to find damping constant for large system. Therefore, modelling of large system with damping present can be done using Rayleigh damping or proportional damping. Here the $[C]$ matrix can be determined through given equation which are given below:

$$\{X\}^T [C]\{X\} = \begin{bmatrix} \alpha + \beta\omega_1^2 & 0 & \cdots & 0 \\ 0 & \alpha + \beta\omega_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha + \beta\omega_N^2 \end{bmatrix} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & \cdots & 0 \\ 0 & 2\xi_2\omega_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 2\xi_N\omega_N \end{bmatrix} \quad (15)$$

Where , $\{X\}$ represents eigenvector of the system, α and β are the coefficients to be determined for N simultaneous equations.

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (16)$$

After assembly of elemental matrices, the equation of motion of whole structure can be represented as

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + [K]\{d\} = \{F\} \quad (17)$$

Chapter 5

Results

and

Discussions

RESULTS AND DISCUSSIONS

This theory has been implemented to a beam with *L/h* =5 to study the convergence of dimensionless fundamental frequency for V_{cnt}^{*} = 0.12.It is calculated by following formula [31]

$$\lambda^2 = \omega L^2 \sqrt{\frac{\rho_m A}{E_m I}} \tag{18}$$

The various material properties of carbon nanotube at different temperatures are provided in Table 1 and different material properties of matrix has been considered as E_m equal to 2.1GPa; v_m equal to 0.34 and ρ_m equal to 1150 Kg/m^3 respectively and the results have been carried out for first six frequencies with end conditions clamped–clamped (C–C) which is non-dimensional, FG-CNTRC beams based on Timoshenko beam theory are obtained for different slenderness ratios, various CNT volume fraction, various CNT distributions and for different temperature distributions. It can be seen that the present result converges by taking numbers of element N equal to 100.The results obtained is in agreement with the results present in the literature.

Using all the above theories, a complete code of MATLAB has been developed. The beam which has been used in this case is simply supported with their end boundary conditions. The cross section of the beam is rectangular with length (L) equal to 0.5m and width (b) equal to 0.4m. The volume fraction of the constituents are varied in the thickness direction. The beam is divided into eleven layers along the thickness. The effective temperature dependent (TD) and temperature independent (TID) material properties of each layer has been determined by using simple power law and extended rule of mixture. The required material properties are given in Tables 1 and 2.Using all above properties shown in the table graph is drawn between young modulus and thickness direction in order to observe the variation of Young's modulus in the thickness direction for temperature-dependent (TD) and temperature-independent(TID) effective material properties. It is shown in Fig. 6. It is easily observed from the graph that as volume fraction of CNT increases in the thickness direction, Young's modulus increases and is higher for temperature independent criterion. The temperature boundary conditions considered here are,

$$T_0 = 300^\circ K, \quad T_m = 700^\circ K, \quad T_{cnt} = 1000^\circ K$$

These boundary conditions are applied for each case. Graph is plotted to observe the temperature distribution along the thickness according to power law. It can be observed in Fig. 5.

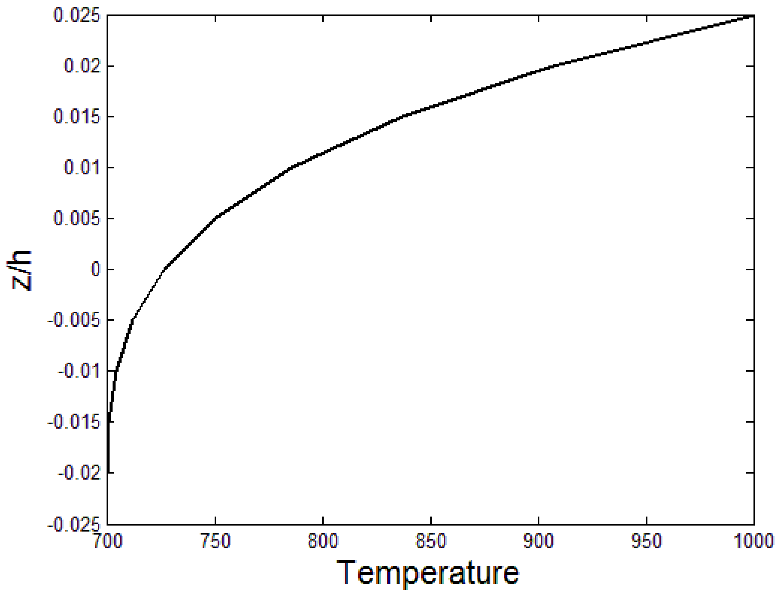

Figure 4. Variation of temperature along the thickness

Table 4: Volume fraction and young's modulus variation along the thickness for various temperature distribution

			Young's Modulus (GPa)	
***T* (K)**	***z/h***	***V_{cnt}***	**Temperature-independent**	**Temperature-dependent**
700	-0.5	0	349.55	304.18
700.09	-0.4	0.32	2184.27	1939.15
701.06	-0.3	0.44	2944.24	2616.24
704.40	-0.2	0.54	3527.38	3135.27
712.07	-0.1	0.63	4018.99	3571.69
726.40	0	0.70	4452.11	3954.03

750.04	0.1	0.77	4843.68	4295.93
785.92	0.2	0.83	5203.76	4603.71
837.22	0.3	0.89	5538.92	4877.88
907.36	0.4	0.94	5853.72	5111.07
1000	0.5	1	6151.45	5281.51

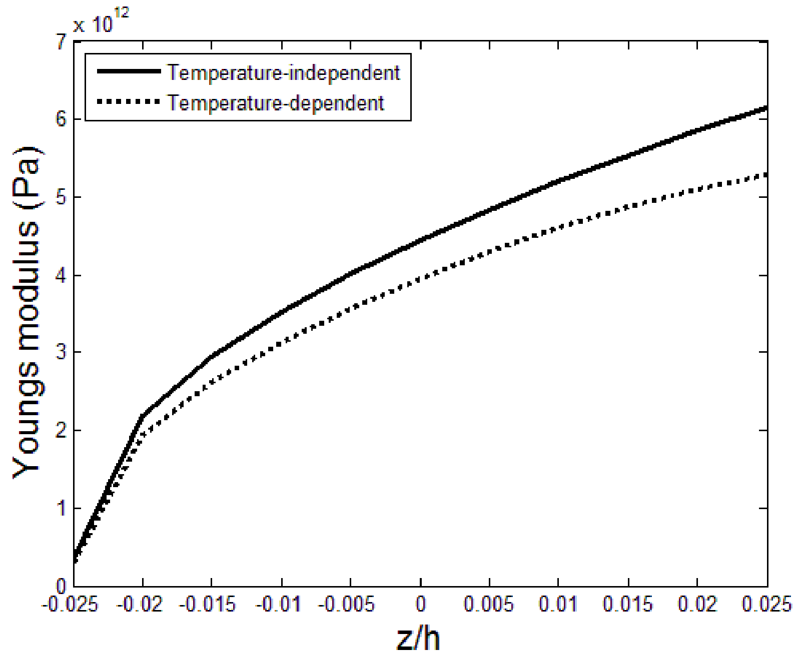

Figure 5. Variation of young modulus in the thickness direction

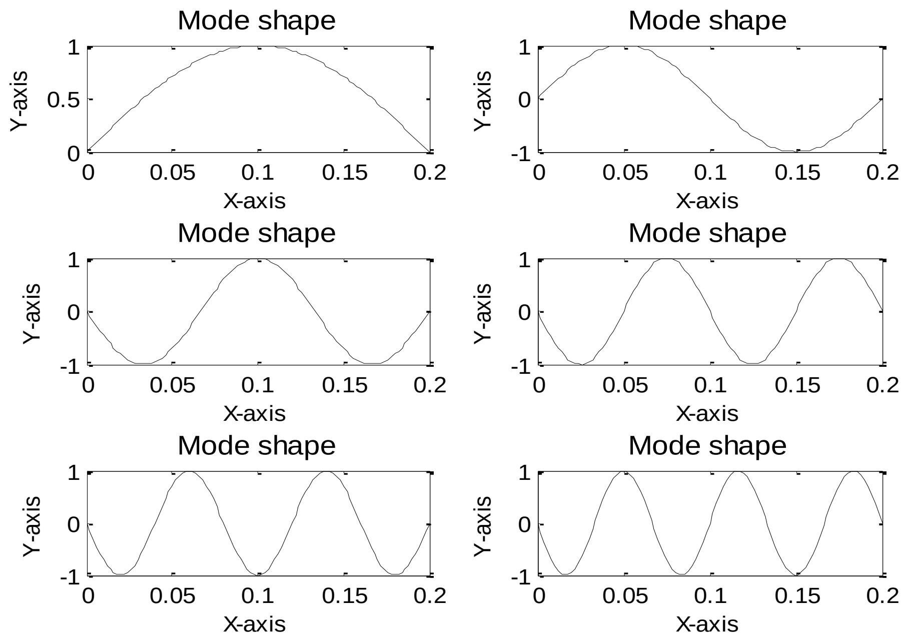

Figure 6. Mode shape for first six fundamental frequency

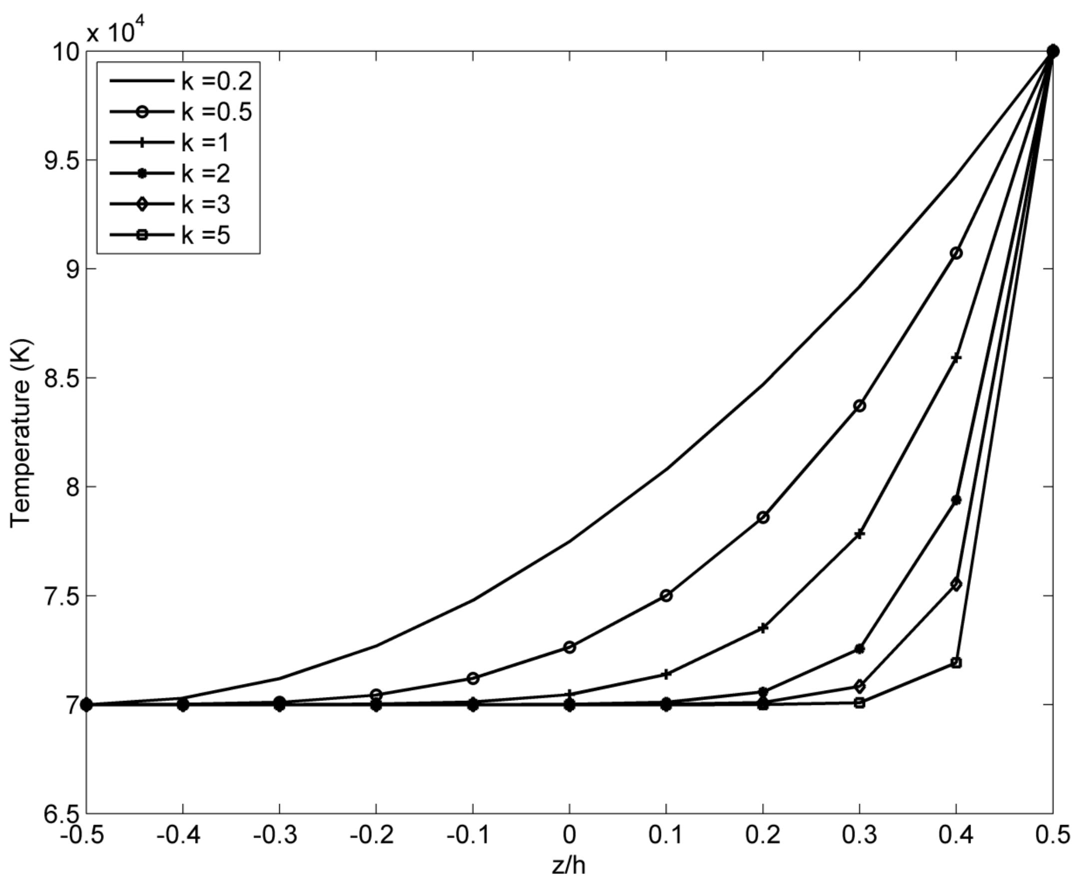

Figure 7. Variation of temperature with z with various power law indexes

h

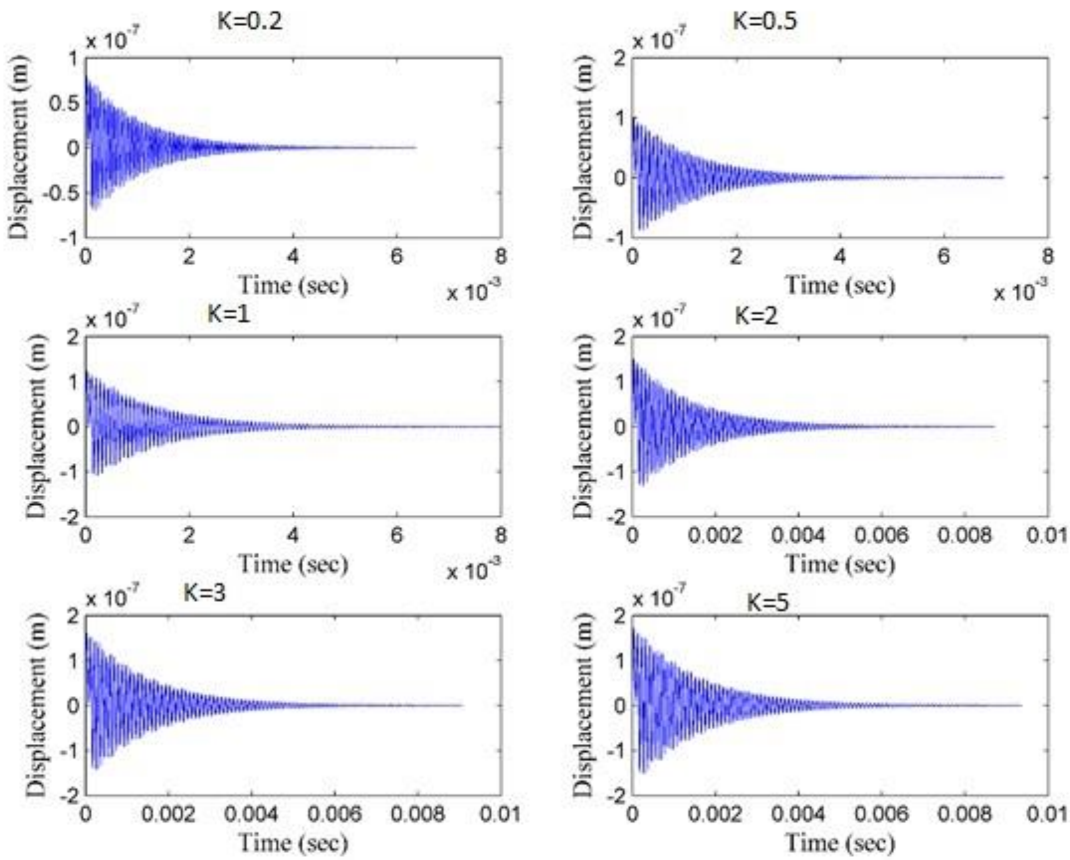

Figure 8. Transient response for various power law indexes

Fig. 4 shows the variation of temperature along the thickness of the beam. The Figure 5 exhibits that with increasing z/h, both temperature dependent and temperature independent young modulus increases but for a particular z/h, temperature independent young modulus is more. The Young's modulus is in the increasing trend layer wise for both temperature-dependent and temperature-independent material properties. Fig 6 shows six mode shape for first six fundamental frequency. The mode shape determines in which mode a system can be excited. Fig 7 shows variation of temperature with z with different power index. As power index increases with same value of z/h the temperature decreases with respect to lower power index .Fig 8 shows that with increase of volume fraction of CNT, the vibration amplitude decreases for different power law index. The result shows that with addition of small amount of CNT fundamental frequency increases. The fundamental frequency of the beam is also influenced by the temperature. Addition of small amount of CNT increases damping.

Chapter 6

Conclusions

and

Scope for Future

Work

CONCLUSIONS AND SCOPE FOR FUTURE WORK

6.1. Conclusions

The present work deals with the vibration and transient response analysis of FG-CNTRC beam for temperature dependent and independent material properties. An attempts has been made to study the effects of temperature on material properties and consequently on vibration behavior of the structure. Based on the analysis the following conclusions can be drawn.

i. Material properties get affected significantly when subjected to high temperatures.

ii. Temperature variation in ceramics is quite insignificant.

iii. Results exhibit that with enhancement in volume fraction of CNT, there is increase in fundamental frequency. The stiffness of the beam increases with enhancement in volume of CNT due to which frequencies are enhanced. The results also reveal that the fundamental frequency of FG-CNTRC beam is significantly influenced by the temperature.

iv. Transient analysis revealed that the various parameters like CNT volume fraction, power law index and TID properties has remarkable effect on vibration.

Thus, higher volume content of CNT in the FGM may yield better dynamic behavior, as higher content of ceramic would lead to more static temperature zones.

6.2. Scope for Future Work

i. Analyses could be carried out to study frequency response of beam structures.

ii. Analyses could be done to determine the optimal value of the power law index that gives the perfect material grading that will give smoother temperature gradation.

iii. Analyses could be carried out to study damping characteristics of beam.

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